

The QCD evolution of TMD in the covariant approach *

A.V. Efremov^a, O.V. Teryaev^a and P. Zavada^b

^a*Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia and*

^b*Institute of Physics AS CR, Na Slovance 2, CZ-182 21 Prague 8, Czech Republic*

The procedure for calculation of the QCD evolution of transverse momentum dependent distributions within the covariant approach is suggested. The standard collinear QCD evolution together with the requirements of relativistic invariance and rotational symmetry of the nucleon in its rest frame represent the basic ingredients of our approach. The obtained results are compared with the predictions of some other approaches.

1. INTRODUCTION

In our previous study we discussed various aspects of the covariant quark-parton model, see [1–6] and citations therein. The discussion included distribution and structure functions (PDF-parton distribution function, TMD-transverse momentum distributions, unpolarized and polarized structure functions) in the leading order. For a fixed Q^2 we obtained the set of relations and rules, which interrelate some of them, for example PDF with TMD:

$$q(x) \rightleftharpoons q(x, p_T), \quad (1)$$

as we have studied in [2, 3].

The aim of the present talk is to discuss a possible procedure for the Q^2 – evolution in the covariant approach. The starting point will be the use of a standard algorithm for the collinear QCD evolution of integrated distribution: $q(x) \rightarrow q(x, Q^2)$. Then, the evolved distribution $q(x, Q^2)$ is inserted into the original relation (1). In this way we have defined the procedure for the evolution of TMD:

$$q(x) \rightarrow q(x, Q^2) \rightarrow q(x, p_T, Q^2). \quad (2)$$

In the present study we will discuss unpolarized distributions only. In the next section we collect corresponding relations defining the procedure (1) in detail. The last section involves the numerical results and its discussion.

2. EVOLUTION OF TMD

The well known algorithm for evolution of the unpolarized PDF reads:

$$\frac{d}{d \ln Q^2} q(x, Q^2) = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) q(x, Q^2), \quad (3)$$

where P is the corresponding splitting function. We have shown [5] that in the leading order approach the relativistic invariance and spherical symmetry in the nucleon rest frame imply

$$\rho_q(p, Q^2) = 4\pi p^2 M G_q(p, Q^2) = -x^2 \left(\frac{q(x, Q^2)}{x} \right)' = q(x, Q^2) - x q'(x, Q^2); \quad p(x) = \frac{Mx}{2}. \quad (4)$$

where $\rho_q(p)$ is the probability distribution of the quark momentum $p = |\mathbf{p}|$ in the rest frame, $f'(x, Q^2)$ denotes df/dx . This relation is valid for any sufficiently large Q^2 , when the quark can be considered effectively free in any frame in the sense of Ref. [1]. The relation (3) implies

$$\frac{d}{d \ln Q^2} q'(x, Q^2) = -\frac{1}{x} P\left(\frac{x}{x}\right) q(x, Q^2) + \int_x^1 \frac{dy}{y} \frac{d}{dx} P\left(\frac{x}{y}\right) q(y, Q^2). \quad (5)$$

Since

$$\frac{d}{dx} P\left(\frac{x}{y}\right) = -\frac{y}{x} \frac{d}{dy} P\left(\frac{x}{y}\right), \quad (6)$$

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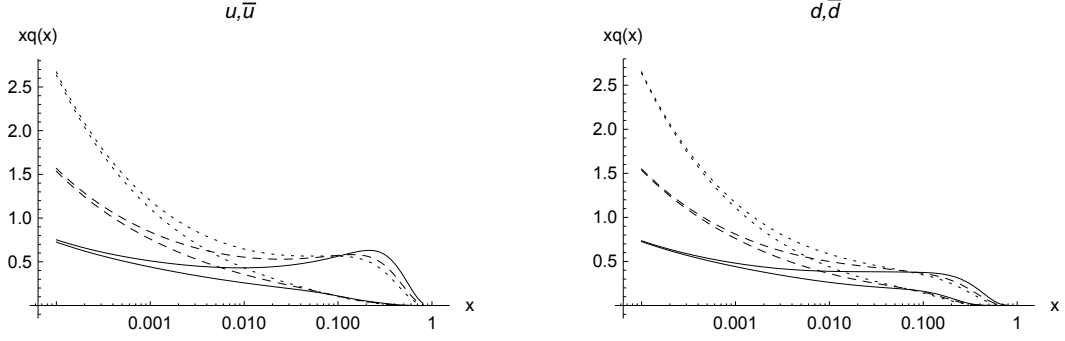


FIG. 1: Input PDF of u, \bar{u} (left) and d, \bar{d} (right) quarks at different scales: $Q^2 = 4, 40, 400 \text{ GeV}$ (solid, dashed, dotted curves)

integration by parts gives

$$\frac{d}{d \ln Q^2} (xq'(x, Q^2)) = \int_x^1 \frac{dy}{y} \frac{d}{dx} P\left(\frac{x}{y}\right) (yq'(y, Q^2)). \quad (7)$$

This equality together with Eqs. (3) and (4) gives

$$\frac{d}{d \ln Q^2} \rho_q(p(x), Q^2) = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \rho_q(p(y), Q^2) \quad (8)$$

or equivalently

$$\frac{d}{d \ln Q^2} G_q(p, Q^2) = \frac{1}{4\pi p^2 M} \int_p^{M/2} \frac{dp'}{p'} P\left(\frac{p}{p'}\right) \rho_q(p', Q^2). \quad (9)$$

Further, in Ref. [4, 5] (see also [7]) we proved the relation for the unpolarized TMD:

$$f_1^q(x, p_T, Q^2) = M G_q(\tilde{p}, Q^2) = \frac{1}{4\pi \tilde{p}^2} (q(\xi, Q^2) - \xi q'(\xi, Q^2)), \quad (10)$$

where

$$\tilde{p}(x, p_T) = \frac{M\xi}{2}, \quad \xi = x \left(1 + \left(\frac{p_T}{Mx}\right)^2\right). \quad (11)$$

Relation (10) with the use of (9) give the TMD evolution:

$$\frac{d}{d \ln Q^2} f_1^q(x, p_T, Q^2) = \frac{1}{4\pi \tilde{p}^2} \int_{\tilde{p}}^{M/2} \frac{dp'}{p'} P\left(\frac{\tilde{p}}{p'}\right) \rho_q(p', Q^2) \quad (12)$$

Note the same splitting function in convolutions (3), (8), (9) and (12), which follows from the fact the distributions $q(x, Q^2)$, $\rho_q(p, Q^2)$ and $f_1^q(x, p_T, Q^2)$ are equivalent, as a result of relativistic invariance and rotational symmetry.

At the same time, in an accordance with Eq. (10), the evolved TMD can be expressed directly by means of the evolved PDF. In the next this relation will be used for numerical calculations of TMD evolution.

3. RESULTS

For numerical calculation we have used the PDF set MSTW2008(LO) [8] at the three different scales, which are displayed in Fig.1. In the next figure (Fig.2) we have shown their representation in terms of the distributions $\rho_q(p, Q^2)$ calculated from the relation (4). Finally, in Fig.3 we have displayed the corresponding TMDs. One can observe two important features:

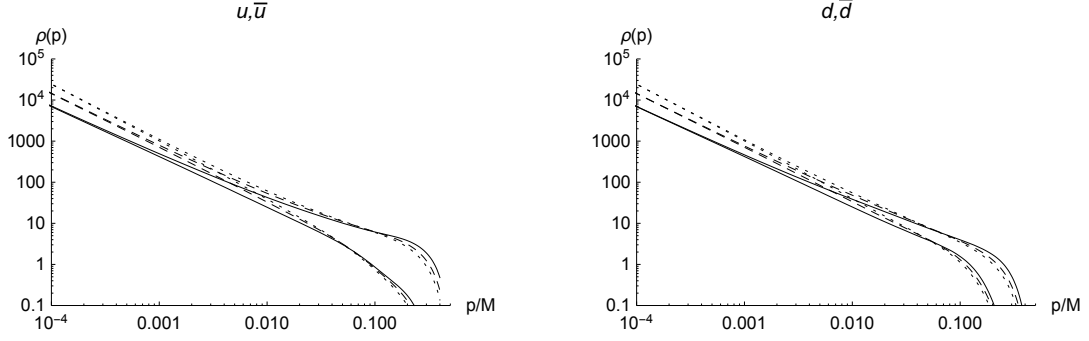


FIG. 2: Distribution of the momentum in the nucleon rest frame for the quarks u, \bar{u} (left) and d, \bar{d} (right) at different scales: $Q^2 = 4, 40, 400 \text{ GeV}$ (solid, dashed, dotted curves)

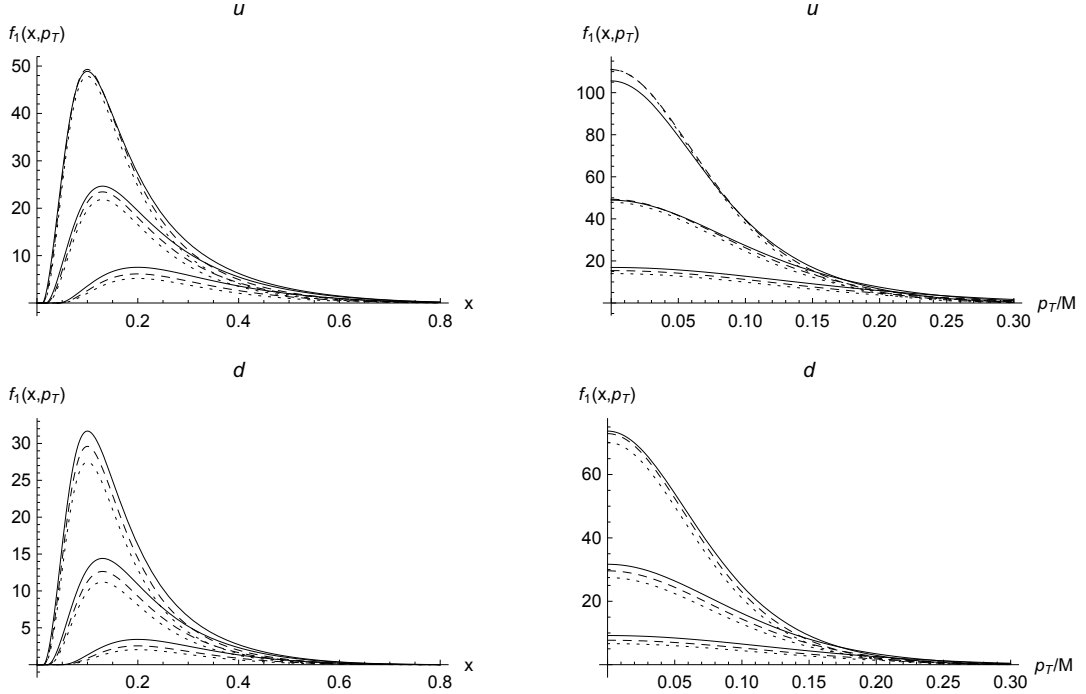


FIG. 3: TMD at different scales: $Q^2 = 4, 40, 400 \text{ GeV}$ (solid, dashed, dotted curves) for u and d quarks. Sets of curves in left panels (from top) correspond to fixed $p_T/M = 0.1, 0.13, 0.20$. The curves in right panels (from top) correspond to fixed $x = 0.18, 0.22, 0.30$.

i) The transverse moments of quarks satisfy the condition $p_T < M/2$. This condition follows from the constraint $0 < x_B < 1$ and the condition of relativistic invariance and rotational symmetry as we have shown in Ref. [6].

ii) Dependence of TMD on the scale Q^2 is rather weak.

These results are well compatible with the predictions obtained within the statistical approach and presented in Ref. [9]. On the other hand our results on TMD differs rather substantially e.g. from the results of the QCD evolution in Ref. [10]. The possible resolution of this contradiction is that evolution considered here may be attributed to the 'soft' non-perturbative component of TMDs and even can be used to disentangle it from the 'hard' perturbative component. Apparently these discrepancies require further study.

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- [1] P. Zavada, Phys. Rev. D **89**, no. 1, 014012 (2014) [arXiv:1307.0699 [hep-ph]].
 - [2] A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, Phys. Rev. D **83**, 054025 (2011) [arXiv:1012.5296 [hep-ph]].
 - [3] A. V. Efremov, P. Schweitzer, O. V. Teryaev and P. Zavada, Phys. Rev. D **80**, 014021 (2009) [arXiv:0903.3490 [hep-ph]].
 - [4] P. Zavada, Phys. Rev. D **83**, 014022 (2011) [arXiv:0908.2316 [hep-ph]].
 - [5] P. Zavada, Eur. Phys. J. C **52**, 121 (2007) [arXiv:0706.2988 [hep-ph]].
 - [6] P. Zavada, Phys. Rev. D **85**, 037501 (2012) [arXiv:1106.5607 [hep-ph]].
 - [7] U. D'Alesio, E. Leader and F. Murgia, Phys. Rev. D **81**, 036010 (2010) [arXiv:0909.5650 [hep-ph]].
 - [8] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C **63**, 189 (2009) [arXiv:0901.0002 [hep-ph]].
 - [9] C. Bourrely, F. Buccella and J. Soffer, Int. J. Mod. Phys. A **28**, 1350026 (2013) [arXiv:1302.4281 [hep-ph]].
 - [10] S. M. Aybat and T. C. Rogers, Phys. Rev. D **83**, 114042 (2011) [arXiv:1101.5057 [hep-ph]].